(C)

Given: ***y" + 5y = 4y'*** . (30)

Solution:

$$y''+5y = 4y'$$

$$0 = y''-4y'+5y$$

$$= (D^2 - 4D + 5)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 - 4D + 5)y = (D^2 - 4D + 5)e^{\lambda t}$$

$$Use \ the \ \text{" slide - rule"}$$

$$0 = e^{\lambda t}(\lambda^2 - \underset{a}{4}\lambda + \underset{b}{5})$$

$$0 = (\lambda^2 - 4\lambda + 5) \quad Characteristic \ eqn. \qquad (38)$$

$$0 = (\lambda ? 5)(\lambda ? 1)$$

Method 1:

$$a^2 - 4b = 16 - 4(5) = -4 > 0$$

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t)$$

$$\alpha = \frac{a}{2} \quad ; \quad \beta = \frac{\sqrt{4b^2 - a^2}}{2}$$

$$\alpha = \frac{4}{2} = 2 \quad ; \quad \beta = \frac{\sqrt{4}}{2} = \frac{2}{2} = 1$$

$$\therefore$$

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) = e^{2t}(c_1 \cos 2t + c_2 \sin 2t)$$

$$y = [c_1 y_1 + c_2 y_2]$$

$$y_1 = e^{2t} \cos t \quad and \quad y_2 = e^{2t} \sin t$$

Or as a solution vector:

$$\bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{2t} \cos t \\ e^{2t} \sin t \end{pmatrix} = e^{2t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

$$y = (y_1, y_2) = (e^{2t} \cos t, e^{2t} \sin t)$$

Method 2: From equation (38)

$$\lambda^2 - 4\lambda + 5 = 0$$

$$\lambda = \frac{4 \pm \sqrt{16 - 4(5)}}{2} = 2 \pm i$$

Since *Euler* yields

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$e^{(2 \pm i)t} = e^{2t} e^{\pm it} = e^{2t}(\cos t \pm i \sin t)$$

we get ℝeal solutions

$$\begin{bmatrix} y_1 = e^{2t} \cos t \\ y_2 = e^{2t} \sin t \end{bmatrix}.$$

(D)

Given: ***y" + 6y = 5y'*** . (60)

Solution:

$$y'' + 6y = 5y'$$

$$0 = y'' - 5y' + 6y$$

$$= (D^2 - 5D + 6)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 - 5D + 6)y = (D^2 - 5D + 6)e^{\lambda t}$$

$$Use \ the \ "slide - rule"$$

$$0 = e^{\lambda t}(\lambda^2 - 5\lambda + 6)$$

$$0 = (\lambda^2 - \underset{a}{5}\,\lambda + \underset{b}{6}) \quad \textit{Characteristic eqn.} \qquad (68)$$

$$0 = (\lambda - 2)(\lambda - 3) \qquad @\ \lambda \in \{2,3\} \equiv \{\lambda_1 \lambda_2\}$$

$$\therefore \quad \begin{bmatrix} y_1 = e^{\lambda_1 t} = e^{2t} \\ y_2 = e^{\lambda_2 t} = e^{3t} \end{bmatrix}$$

$$y = \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{2t} \\ e^{3t} \end{pmatrix} = (e^{2t}, e^{3t}) = (y_1, y_2)$$

(E)

Given: ***y” + 4y = 4y’*** . (70)

Solution:

$$y'' + 4y = 4y'$$

$$0 = y'' - 4y' + 4y$$

$$= (D^2 - 4D + 4)y$$

$$\textit{Let} \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 - 4D + 4)y = (D^2 - 4D + 4)e^{\lambda t}$$

$$\textit{Use the "slide} - \textit{rule"}$$

$$0 = e^{\lambda t}(\lambda^2 - 4\lambda + 4)$$

$$0 = (\lambda^2 - 4\lambda + 4) \quad \textit{Characteristic eqn.} \qquad (78)$$

$$0 = (\lambda - 2)(\lambda - 2) \qquad @\ \lambda \in \{2,2\}$$

$$\textit{Double root}$$

$$\therefore \quad y_1 = e^{\lambda_1 t} \quad \cup \quad y_2 = e^{\lambda_2 t} = te^{\lambda_1 t}$$

Or as a solution vector:

$$\bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{2t} \\ te^{2t} \end{pmatrix} = e^{2t}\begin{pmatrix} 1 \\ t \end{pmatrix}$$

$$y = (y_1, y_2) = (e^{2t}, te^{2t}) = e^{2t}(1, t)$$

(F)

Given: ***y" + 50y = 2y'*** . (90)

Solution:

$$y'' + 5y = 4y'$$

$$0 = y'' - 4y' + 5y$$

$$= (D^2 - 4D + 5)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 - 4D + 5)y = (D^2 - 4D + 5)e^{\lambda t}$$

$$Use \ the \ " slide - rule"$$

$$0 = e^{\lambda t}(\lambda^2 - \underset{a}{4}\lambda + \underset{b}{5})$$

$$0 = (\lambda^2 - 4\lambda + 5) \quad Characteristic \ eqn. \qquad (98)$$

Since $a^2 - 4b = 4 - 200 = -196 < 0$ there are 2 complex solutions. The solutions can be resolved from
method 1:

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) \qquad (100)$$

$$where \quad \alpha = -\frac{a}{2} = 1 \quad ;$$

$$\beta = \frac{\sqrt{4b - a^2}}{2} = \frac{\sqrt{196}}{2} = 7$$

$$\therefore \ eqn\,(100) \ becomes$$

$$y = c_1 y_1 + c_2 y_2 = e^t (c_1 \cos 7t + c_2 \sin 7t)$$

$$\begin{bmatrix} y_1 = e^t \cos 7t \\ y_2 = e^t \sin 7t \end{bmatrix}$$

$$Or \qquad \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^t \cos 7t \\ e^t \sin 7t \end{pmatrix} = e^t \begin{pmatrix} \cos 7t \\ \sin 7t \end{pmatrix}$$

Method 2: From eqn (98)

$$\lambda^2 - 2\lambda + 50 = 0$$

$$\lambda = \frac{2 \pm \sqrt{-196}}{2} = \frac{2 \pm 14i}{2} = 1 \pm 7i$$

$$The \;\; Euler \;\; eqn \;\; yields$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$e^{(1 \pm 7i)t} = e^t e^{\pm 7it} = e^t (\cos 7t \pm i \sin 7t)$$

$$Which \;\; yields \;\; \mathrm{Re}(Solutions)$$

$$y_1 = e^t \cos 7t \quad \cup \quad y_2 = e^t \sin 7t$$

Done

(G)

Given: ***y" + 6y' + 10y = 0*** . (110)

Solution:

$$y''+6y'+10y = 0$$

$$0 = y''+6y'+10y$$

$$= (D^2 + 6D + 10)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 + 6D + 10)y = (D^2 + 6D + 10)e^{\lambda t}$$

$$Use \quad the \quad "slide - rule"$$

$$0 = e^{\lambda t}(\lambda^2 + 6\lambda + 10)$$

$$0 = (\lambda^2 + \underset{a}{6}\,\lambda + \underset{b}{10}) \quad Characteristic \quad eqn. \qquad (118)$$

Since $a^2 - 4b = 36 - 40 = -4 < 0$ there are 2 complex solutions. The solutions can be resolved from

method 1:

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) \qquad (120)$$

$$where \quad \alpha = -\frac{a}{2} = -3$$

$$\beta = \frac{\sqrt{4b - a^2}}{2} = \frac{2}{2} = 1$$

$$\therefore \; eqn\,(120) \;\; becomes$$

$$y = c_1 y_1 + c_2 y_2 = e^{-3t}(c_1 \cos t + c_2 \sin t)$$

$$\left[\quad y_1 = e^{-3t} \cos t \qquad y_2 = e^{-3t} \sin t \right]$$

$$Or \quad \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{-3t} \cos t \\ e^{-3t} \sin t \end{pmatrix} = e^{-3t}\begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Method 2: From eqn(118)

$$\lambda^2 + 6\lambda + 10 = 0$$

$$\lambda = \frac{-6 \pm \sqrt{36 - 40}}{2} = \frac{-6 \pm 2i}{2} = -3 \pm i$$

The Euler eqn yields

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{(-3\pm i)t} = e^{-3t}e^{\pm it} = e^{-3t}(\cos t \pm i\sin t)$$

Which yields $\text{Re}(Solutions)$

$$y_1 = e^{-3t}\cos t \quad \cup \quad y_2 = e^{-3t}\sin t$$

(H)

Given: ***y" + 10y' + 29y = 0*** . (130)

Solution:

$$y''+10y'+29y = 0$$

$$0 = y''+10y'+29y$$

$$= (D^2 + 10D + 29)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 + 10D + 29)y = (D^2 + 10D + 29)e^{\lambda t}$$

Use the " slide − rule"

$$0 = e^{\lambda t}(\lambda^2 + 10\lambda + 29)$$

$$0 = (\lambda^2 + \underset{a}{10}\,\lambda + \underset{b}{29}) \quad Characteristic\ eqn. \qquad (138)$$

Since $a^2 - 4b = 100 - 116 = -16 < 0$ there are 2 complex solutions. The solutions can be resolved from

method 1:

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) \qquad (140)$$

$$where \quad \alpha = -\frac{a}{2} = -\frac{10}{2} = -5$$

$$\beta = \frac{\sqrt{4b - a^2}}{2} = \frac{4}{2} = 2$$

$$\therefore \; eqn\,(140) \; becomes$$

$$y = c_1 y_1 + c_2 y_2 = e^{-5t}(c_1 \cos 2t + c_2 \sin 2t)$$

$$\left[y_1 = e^{-5t} \cos 2t \quad \cup \quad y_2 = e^{-5t} \sin 2t \right]$$

$$Or \quad \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{-5t} \cos 2t \\ e^{-5t} \sin 2t \end{pmatrix} = e^{-5t}\begin{pmatrix} \cos 2t \\ \sin 2t \end{pmatrix}$$

Method 2: From eqn(138)

$$\lambda^2 + 10\lambda + 29 = 0$$

$$\lambda = \frac{-10 \pm \sqrt{100 - 116}}{2} = \frac{-10 \pm \sqrt{-16}}{2}$$

$$\lambda = \frac{-10 \pm 4i}{2} = -5 \pm 2i$$

$$The \quad Euler \quad eqn \quad yields$$

$$e^{i\theta} = \cos\theta + i \sin\theta$$

$$e^{(-5 \pm 2i)t} = e^{-5t} e^{\pm 2it} = e^{-5t}(\cos 2t \pm i \sin 2t)$$

$$Which \quad yields \; \mathrm{Re}(Solutions)$$

$$y_1 = e^{-5t} \cos 2t \quad \cup \quad y_2 = e^{-5t} \sin 2t$$

Done

(C)

Given: ***y” + 12y’ + 35y = 0*** . (160)

Solution:

$$y''+12y'+35y = 0$$
$$0 = y''+12y'+35y$$
$$= (D^2 + 12D + 35)y$$
$$Let \quad y = e^{\lambda t} = e^{st}$$
$$\therefore$$
$$0 = (D^2 + 12D + 35)y = (D^2 + 12D + 35)e^{\lambda t}$$
$$Use \quad the \quad \text{" } slide - rule \text{"}$$
$$0 = e^{\lambda t}(\lambda^2 + 1\lambda 2 + 35)$$
$$0 = (\lambda^2 + 12\lambda + 35) \quad Characteristic \quad eqn. \qquad (168)$$
$$0 = (\lambda + 7)(\lambda + 5)$$

$$\therefore \quad @ \quad \lambda \in \{\lambda_1 \lambda_2\} \equiv \{-5,-7\}$$
$$\begin{bmatrix} y_1 = e^{\lambda_1 t} = e^{-5t} \\ y_1 = e^{\lambda_2 t} = e^{-7t} \end{bmatrix}$$

Or as a solution vector:

$$\bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{-5t} \\ e^{-7t} \end{pmatrix}$$
$$y = (y_1, y_2) = (e^{-5t}, e^{-7t})$$

Done

This problem pertains to the equation

$$y''+4y'+ay = 0 \tag{180}$$

where ***a*** is a ℝeal constant. Obtain the following particular solutions:

$$e^{-t} \quad ; \quad te^{-2t} \quad ; \quad e^{-2t}\cos t \tag{182}$$

for ***a* = *3, 4, 5*** respectively.

(A)

Solution:

$$y''+4y'+ay = 0$$

$$0 = y''+4y'+ay$$

$$= (D^2 + 4D + a)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$$\therefore$$

$$0 = (D^2 + 4D + a)y = (D^2 + 4D + a)e^{\lambda t}$$

$$Use \ the \ "slide - rule"$$

$$0 = e^{\lambda t}(\lambda^2 + 4\lambda + a)$$

$$0 = (\lambda^2 + \underset{A}{4}\lambda + \underset{B}{a}) \quad Characteristic \ eqn. \tag{188}$$

$$Since \quad A^2 - 4B = 16 - 4a = ? \quad Then \ for \ a = 3$$

$$16 - 4a = 16 - 12 = 4 > 0$$

$$Yields \ \mathrm{Re}(Solutions).$$

$$Thus \quad \lambda^2 + 4\lambda + a = \lambda^2 + 4\lambda + 3 = 0$$

$$(s + 3)(s + 1) = 0 \quad @ \quad \lambda \in \{\lambda_1 \lambda_2\} \equiv \{-1,-3\}$$

$$\begin{bmatrix} y_1 = e^{\lambda_1 t} = e^{-t} \\ y_2 = e^{\lambda_2 t} = e^{-3t} \end{bmatrix}$$

(B)

For ***a = 4***

$$\lambda^2 + 4\lambda + a = \lambda^2 + 4\lambda + 4 = 0$$

$$(\lambda + 2)(\lambda + 2) = 0$$

$$@ \quad \lambda = -2. \quad \therefore \; Double \;\; root \;\; @ \, \lambda_1 = \lambda_2 = -2$$

$$Thus \quad y_1 = e^{\lambda_1 t} \quad \cup \quad y_2 = e^{\lambda_2 t} = te^{\lambda_1 t}$$

$$So \; that \begin{bmatrix} y_1 = e^{-2t} \\ y_2 = te^{-2t} \end{bmatrix}$$

(C)

For ***a = 5***

$$A^2 - 4B = ?$$

$$\lambda^2 + 4\lambda + a = \lambda^2 + \underset{A}{4}\,\lambda + \underset{B}{5} = 0 \qquad (208)$$

Since $A^2 - 4B = 16 - 4(5) = -4 < 0$ there are 2 complex solutions. The solutions can be resolved from
method 1:

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) \qquad (210)$$

$$where \quad \alpha = -\frac{a}{2} = -\frac{A}{2} = -\frac{4}{2} = -2$$

$$\beta = \frac{\sqrt{4B - A^2}}{2} = \frac{2}{2} = 1$$

$$\therefore \; eqn\,(210) \;\; becomes$$

$$y = c_1 y_1 + c_2 y_2 = e^{-2t}(c_1 \cos t + c_2 \sin t)$$

$$\left[\quad y_1 = e^{-2t} \cos t \qquad y_2 = e^{-2t} \sin t \right]$$

$$Or \quad \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{-2t} \cos t \\ e^{-2t} \sin t \end{pmatrix} = e^{-2t} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

Method 2: From eqn(208)

$$\lambda^2 + 10\lambda + 29 = 0$$

$$\lambda = \frac{-10 \pm \sqrt{100 - 116}}{2} = \frac{-10 \pm \sqrt{-16}}{2}$$

$$\lambda = \frac{-10 \pm 4i}{2} = -5 \pm 2i$$

The Euler eqn yields

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{(-5\pm 2i)t} = e^{-5t}e^{\pm 2it} = e^{-5t}(\cos 2t \pm i\sin 2t)$$

Which yields Re(*Solutions*)

$$y_1 = e^{-5t}\cos 2t \quad \cup \quad y_2 = e^{-5t}\sin 2t$$

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Parts a, c
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This problem pertains to the equation

$$y''+2by'+2y = 0 \qquad (1)$$

where ***b*** is a ℝeal constant. For $b^2 = 1, 2, 3$ respectively, obtain the following particular solutions:

$$e^{-bt}\cos t \quad ; \quad te^{-bt} \quad ; \quad e^{-bt}e^{t} \qquad (2)$$

(A)
Solution:

$$y''+2by'+2y = 0$$

$$0 = y''+2by'+2y$$

$$= (D^2 + 2bD + 2)y$$

$$Let \quad y = e^{\lambda t} = e^{st}$$

$\therefore$

$$0 = (D^2 + 2bD + 2)y = (D^2 + 2bD + 2)e^{\lambda t}$$

" slide it baby"

$$0 = e^{\lambda t}(\lambda^2 + 2b\lambda + 2)$$

$$0 = (\lambda^2 + \underset{A}{2b}\lambda + \underset{B}{2}) \quad \textit{Characteristic eqn.} \qquad (8)$$

$$\textit{If } A^2 - 4B \ \textit{ Then for } b = 1$$

$$A^2 - 4B = 4b^2 - 8 = 4 - 8 = -4 < 0$$

Since $A^2 - 4B = -4 < 0$ there are 2 complex solutions. The solutions can be resolved from

method 1:

$$y = e^{\alpha t}(c_1 \cos \beta t + c_2 \sin \beta t) \qquad (10)$$

$$\textit{where} \quad \alpha = -\frac{A}{2} = -\frac{2b}{2} = -b$$

$$\beta = \frac{\sqrt{4B - A^2}}{2} = \frac{\sqrt{8 - 4b^2}}{2} = \frac{2}{2} = 1$$

$$\therefore \ \textit{eqn } (10) \ \textit{ becomes}$$

$$y = c_1 y_1 + c_2 y_2 = e^{-bt}(c_1 \cos t + c_2 \sin t)$$

$$\left[\quad y_1 = e^{-bt} \cos t \quad \cup \quad y_2 = e^{-bt} \sin t \right]$$

$$\textit{Or} \quad \bar{\varphi}(t) = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} e^{-bt} \cos t \\ e^{-bt} \sin t \end{pmatrix} = e^{-bt} \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$$

b = 1

Method 2: From eqn(8)

$$\lambda^2 + 2b\lambda + 2 = 0 \qquad @ \; b^2 = 1$$

$$\lambda = \frac{-2b \pm \sqrt{2^2 b^2 - 8}}{2} = \frac{-2b \pm \sqrt{-4}}{2}$$

$$\lambda = -b \pm i$$

The Euler eqn yields

$$e^{i\theta} = \cos\theta + i\sin\theta$$

$$e^{(-b \pm i)t} = e^{-bt} e^{\pm it} = e^{-bt}(\cos t \pm i \sin t)$$

Which yields Re(*Solutions*)

$$y_1 = e^{-bt}\cos t \quad \cup \quad y_2 = e^{-bt}\sin t$$

(B)

For

$$b^2 = 2 \quad and \quad \lambda^2 + \underset{A}{2b}\lambda + \underset{B}{2} = 0$$

$$A^2 - 4B = 4b^2 - 8 = 8 - 8 = 0$$

There is a double root for $y = e^{\lambda t}$.

Therefore,

$$\lambda^2 + 2b\lambda + 2 = \lambda^2 + 2\sqrt{2}\lambda + 2 = 0$$

$$= (\lambda + \sqrt{2})(\lambda + \sqrt{2}) = 0$$

$$@ \; \lambda \in \{\lambda_1 \lambda_2\} \equiv \{-\sqrt{2}, -\sqrt{2}\}$$

$$\lambda = -b$$

$$\begin{bmatrix} y_1 = e^{\lambda_1 t} = e^{-bt} \\ y_2 = e^{\lambda_2 t} = te^{\lambda_1 t} = te^{-bt} \end{bmatrix}$$

(C)

For

$$b^2 = 3 \quad and \quad \lambda^2 + \underset{A}{2b}\lambda + \underset{B}{2} = 0$$

$$A^2 - 4B = 4b^2 - 8 = 4(3) - 8 = 4 > 0$$

There are 2 ℝeal roots for $y = e^{\lambda t}$.

Therefore,

$$\lambda^2 + 2\sqrt{3}\lambda + 2 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2b \pm \sqrt{4b^2 - 4(2)}}{2}$$

$$\lambda = \frac{-2b \pm \sqrt{12 - 8}}{2} = \frac{-2b \pm 2}{2} = -b \pm 1$$

$\therefore$

$$y = e^{\lambda t} = e^{(-b \pm 1)t} = e^{-bt}e^{\pm t}$$

$$\begin{bmatrix} y_1 = e^{-bt}e^{t} \\ y_2 = e^{-bt}e^{-t} \end{bmatrix}$$

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Get a particular solution of equations {1,2,3} in the simplest possible form and thus get a solution of the 4th equation where ***a, b, c*** are any constants:

$$y'' - y' + 7y = \cos 2t \qquad (1)$$

$$y'' - y' + 7y = \sin 2t \qquad (2)$$

$$y'' - y' + 7y = e^{2t} \qquad (3)$$

$$y'' - y' + 7y = 13(a\cos 2t + b\sin 2t) + 9ce^{2t} \qquad (4)$$

Solution:

Equations (1) and (2) can be represented by

$$z'' - z' + 7z = e^{2it}$$

$$Let \quad \begin{bmatrix} z = Ae^{2it} \\ z' = 2iAe^{2it} \\ z'' = -4Ae^{2it} \end{bmatrix}$$

$$So \quad y'' \rightarrow z'' \quad here.$$

$$e^{2it} = z'' - z' + 7z$$

$$= -4Ae^{2it} - 2iAe^{2it} + 7Ae^{2it}$$

$$1 = -4A - 2iA + 7A = A(-4 - 2i + 7) = A(3 - 2i)$$

$$A = \frac{1}{3 - 2i} \cdot \left(\frac{3 + 2i}{3 + 2i}\right) = \frac{3 + 2i}{9 + 6i - 6i + 4}$$

$$A = \frac{3 + 2i}{13}$$

$$\therefore$$

$$z = Ae^{2it} = \frac{(3 + 2i)}{13} e^{2it} = \frac{(3 + 2i)}{13}[\cos 2t + i \sin 2t]$$

$$= \tfrac{1}{13}[3 \cos 2t + 3i \sin 2t + 2i \cos 2t - 2 \sin 2t]$$

$$z = \tfrac{1}{13}[3 \cos 2t - 2 \sin 2t + i(2 \cos 2t + 3 \sin 2t)]$$

$$So, \quad \begin{bmatrix} y_1 = \tfrac{1}{13}[3 \cos 2t - 2 \sin 2t] \\ y_2 = \tfrac{1}{13}[2 \cos 2t + 3 \sin 2t] \end{bmatrix}$$

Solve equation (3) as follows:

$$y'' - y' + 7y = e^{2t}$$

$$Let \quad y = Ae^{\lambda t} = Ae^{2t}$$

Where $2 \neq$ root of the homogeneous equation.

$$\text{Thus}\quad \begin{bmatrix} y = Ae^{2t} \\ y' = 2Ae^{2t} \\ y'' = 4Ae^{2t} \end{bmatrix}$$

$$\therefore$$

$$e^{2t} = y'' - y' + 7y$$

$$= 4Ae^{2t} - 2Ae^{2t} + 7Ae^{2t}$$

$$1 = 4A - 2A + 7A = 9A$$

$$A = \tfrac{1}{9}$$

$$\text{Then}\quad \left[\quad y_3 = Ae^{2t} = \tfrac{1}{9}e^{2t} \quad\right]$$

Now for the 4th equation do as follows:

$$\text{Let}$$

$$Ty = y'' - y' + 7y = 13(a\cos 2t + b\sin 2t) + 9ce^{2t}$$

By the principle of superposition,

$$Ty_{11} = y'' - y' + 7y = 13a\cos 2t$$

$$Ty_{22} = y'' - y' + 7y = 13b\sin 2t$$

$$Ty_{33} = y'' - y' + 7y = 9ce^{2t}$$

Since equations {1,2,3} are

$$Ty_1 = y'' - y' + 7y = \cos 2t$$

$$Ty_2 = y'' - y' + 7y = \sin 2t$$

$$Ty_1 = y'' - y' + 7y = e^{2t}$$

$$\text{Then}$$

$$Ty_{11} = 13a\cos 2t = 13aTy_1$$

$$y_{11} = 13ay_1 = 13a\left(\tfrac{1}{13}[3\cos 2t - 2\sin 2t]\right)$$

$$\left[\quad y_{11} = a(3\cos 2t - 2\sin 2t \quad \textit{Solution } 1 \quad\right]$$

$$Ty_{22} = 13b\sin 2t = 13b(Ty_2)$$

$$y_{22} = 13by_2 = 13b\left[\tfrac{1}{13}(2\cos 2t + 3\sin 2t)\right]$$

$$\left[\quad y_{22} = b(2\cos 2t + 3\sin 2t) \quad\right] \quad 2^{nd} \quad \textit{Solution}$$

$$Ty_{33} = 9ce^{2t} = 9cTy_3 = T(9cy_3)$$

$$y_{33} = 9cy_3 = \left[\quad 9c\left(\tfrac{1}{9}e^{2t}\right) = ce^{2t} \quad\right] \; 3^{rd} \quad \textit{Solution}$$

Finally,

$$y = y_{11} + y_{22} + y_{33} \qquad \textit{Because}$$

$$Ty = Ty_{11} + Ty_{22} + Ty_{33}$$

$$y = a(3\cos 2t - 2\sin 2t) + b(\cos 2t + 3\sin 2t) + ce^{2t}$$

This is obtained by the principle of superposition.

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This time you are to get a particular solution for each of the two equations (1) & (2):

$$x'' + 2x' + 2x = 2e^{-t}\cos t \tag{1}$$

$$y'' + 2y' + 2y = 2e^{-t}\sin t \tag{2}$$

Try either

① $te^{-t}(A\cos t + B\sin t)$ *or*

② $z = ue^{(-1+i)t}$

If we choose ② , then set $z = ue^{(-1+i)t}$ and use the *slide-rule* with

k = -1 + i in the equivalent complex equation:

$$(D+1+i)(D+1-i)z = 2e^{(-1+i)t} \qquad (5)$$

Note:

$$(D+1+i)(D+1-i)z =$$

$$= (D^2 + D - iD + D + 1 - i + iD + i - i^2)z$$

$$= (D^2 + 2D + 2)z = z''+2z'+2z$$

$$\therefore \quad z''+2z'+2z = 2e^{(-1+i)t}$$

So, from (5)

$$2e^{(-1+i)t} = (D+1+i)(D+1-i)z$$

$$= (D+1+i)(D+1-i)ue^{(-1+i)t}$$

$$= e^{(-1+i)t}(D-1+i+1+i)(D-1+i+1-i)u$$

$$2 = (D+2i)(Du)$$

Logic dictates that ***Du = c = a constant.***

(D + 2i)Du = 2

(D + 2i)c = Dc + 2ic = 2

2ic = 2

$$Thus, \quad c = \frac{2}{2i} = \frac{1}{i} = -i$$

$$\therefore \quad Du = c = -i$$

$$\frac{du}{dt} = i$$

$$\int du = \int -\, idt$$

$$u = -it$$

$$z = ue^{(-1+i)t} = ite^{-t}e^{it} = -ite^{-t}(\cos t + i\sin t)$$

$$z = te^{-t}[\sin t - i\cos t]$$

$$\text{Re } part \rightarrow \left[x = te^{-t}\sin t\right];\ \text{Im } part \rightarrow \left[y = -te^{-t}\cos t\right]$$